Passive time-domain macromodeling of large complex interconnects

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Abstract: This paper is focused on the macromodeling of complex interconnect structures characterized by many ports. The structures are considered as a black-box known via time-domain port responses, which can be obtained through full-wave simulation using a transient field solver. The generation of a passive macromodel is based on the following steps. First, the port responses are split into separate subsets to allow for subsequent concurrent processing. This may required by the possibly large amount of data needed to fully characterize the structure. Second, each subset is processed by a Time-Domain Vector Fitting algorithm providing a rational approximation of the corresponding transfer matrix entries. Third, the various sub-models are assembled into a global sparse state-space realization. Fourth, the passivity of the resulting global macromodel is tested and enforced by a spectral perturbation of associated Hamiltonian matrices. Finally, an equivalent circuit is syntesized directly from the passive spate-space macromodel. The high degree of accuracy of the proposed method is illustrated by some applications.

1 Introduction

It is well known that the signal integrity of electronic systems may be strongly affected by discontinuities in the signal propagation paths. Such discontinuities can be vias and via arrays, bends, junctions, connectors, etc. A careful assessment requires a proper modeling of such structures in order to be able to reproduce their effects on the signals. This modeling procedure should take into account geometry and material properties, thus requiring a complex full-wave electromagnetic analysis. Since a global full-wave analysis of an entire elelectronic system including terminations and hence nonlinearities is not feasible, a common practice is to translate the results of the electromagnetic simulation into a macromodel or an equivalent circuit of reduced complexity, which can be used subsequently for system-level analyses performed via standard circuit solvers like SPICE.

Several approaches are available in the literature for this task. In this paper we present a new methodology that allows to break the complexity of the macromodeling procedure in separate and well-defined steps. The technique is based on the knowledge of all transient responses at the ports of the structure under investigation under suitable port excitations. These can be in the form of transient scattering waves, but the method can be extended also to other representations (open-circuit, short-circuit, or hybrid). All these responses, which are assumed here to be obtained via transient full-wave simulation through, e.g., Finite-Difference Time-Domain, Finite Integration, or transient Partial Element Equivalent Circuit methods, may constitute a very large dataset. Therefore, the first problem to be addressed is the large amount of data to be processed to obtain a global macromodel. For this reason, the proposed methodology is based on a divide-and-conquer method that breaks the initial dataset into disjoint subsets of port responses, which are processed separately. This approach is well-suited to parallelization and to adaptation to grid computing.

The macromodeling procedure is split into the following steps. First, based on the complexity and the number of the port responses, the above described port responses splitting is applied to generate the separate subsets. Each subset corresponds to a disjoint set of matrix entries of the structure transfer function. Second, each subset is processed by a Time-Domain Vector Fitting algorithm [2] providing a rational approximation of the corresponding transfer matrix entries. This rational approximation is constructed by solving only linear least squares problems combined with recursive digital filtering of the port excitations and responses. Third, the various rational sub-models are assembled into a global sparse state-space realization. Fourth, the passivity of the resulting global macromodel is tested using the spectral properties of some Hamiltonian matrices associated to the global state-space realization [4, 5]. A passivity violation is related to the presence of purely imaginary eigenvalue of this matrices. If a passivity violation occurs, an iterative spectral perturbation is applied in order to compensate it. This procedure is also based on a sequence of linear least squares solutions which perturb the original macromodel in order to reach passivity with the minimal impact on the accuracy. Finally, an equivalent circuit is syntesized directly from the passive spate-space macromodel. This is a standard procedure, which is not further discussed here. The other macromodeling steps are described in sections 2–5. Section 6 presents some numerical results.

2 Splitting Port Responses

We consider a multiport structure with an arbitrary number P of ports. Input and output vectors are denoted as $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively. Usually, a transient characterization of such a multiport structure is obtained by exciting one port at the time and computing/measuring the responses at all ports. As a result, the raw data set is a matrix of response waveforms $y_{ij}(t)$ at port (i), due to an excitation source $x_j(t)$ located at port (j). We remark that this type of data set is the natural outcome of time-domain full-wave electromagnetic solvers. Transient scattering waveforms are the typical format, although present formulation is applicable also to transient impedance (open-circuit), admittance (short-circuit), or hybrid representations. The objective is the derivation of a rational approximation to the matrix transfer function $\boldsymbol{H}(s)$, where s is the Laplace variable. This approximation reads, in terms of poles and residues,

$$\boldsymbol{H}(s) \simeq \boldsymbol{H}_\infty + \sum_{n=1}^{N} \frac{\boldsymbol{R}_n}{s - p_n} \,. \tag{1}$$

The raw transient responses are then assumed to satisfy the relation

$$y_{ij}(t) \simeq \mathcal{L}^{-1}\{H_{ij}(s)\} * x_j(t)\,, \quad i,j = 1,\ldots,P\,, \tag{2}$$

where $\mathcal{L}^{-1}$ is the inverse Laplace operator and $*$ denotes convolution.

The generation of the subsets of the port responses is performed using the following procedure. Let

$$\Omega = \{(i,j),\, i,j = 1,\ldots,P\} \tag{3}$$

denote all pairs of available port responses for the given structure. This set is partitioned as

$$\Omega = \bigcup_k \Omega_k\,, \qquad \bigcap_k \Omega_k = \emptyset\,, \tag{4}$$

where

$$\Omega_k = \{(i_\nu, j_k),\, \nu = 1,\ldots,P_k\}\,. \tag{5}$$

Each subset Ω_k is therefore characterized by a fixed excitation (corresponding to the index j_k) and by multiple outputs indexed by i_ν. Although more general partitions can be performed, this choice will lead to a final global macromodel characterized by a minimal number of internal states. Note that if $P_k = P$ for all k, this partition corresponds to a columnwise partition of the transfer matrix. With the above notations we can collect the input and output responses of each subset as

$$x_{j_k} = \boldsymbol{P}_k \boldsymbol{x}\,, \qquad \boldsymbol{y}_k = \boldsymbol{Q}_k \boldsymbol{y}\,, \tag{6}$$

where $\boldsymbol{P}_k$ is a row vector with all vanishing entries except a single unitary entry at location j_k, and $\boldsymbol{Q}_k$ is a $P_k \times P$ selector matrix having a single unitary entry in each row ν at column i_ν. In the following we will look for a partial macromodel for the corresponding subset of the transfer matrix entries defined as the following single-input multiple-output system

$$\boldsymbol{Y}_k(s) = \boldsymbol{H}_k(s) X_{j_k}(s)\,. \tag{7}$$

This partial macromodel reads

$$\boldsymbol{H}_k(s) \simeq \boldsymbol{H}_{k,\infty} + \sum_{n=1}^{N_k} \frac{\boldsymbol{R}_{k,n}}{s - p_{k,n}}\,. \tag{8}$$

3 Time-Domain Vector Fitting

In this section we briefly describe the Time-Domain Vector Fitting algorithm for the identification of the poles/residues terms in Eq. (8). More details can be found in [2, 3]. The TDVF method is a time-domain reformulation of the well-known Vector Fitting (VF) method [6], which operates in frequency domain. This time-domain formulation may be necessary when only transient data are available and no discrete Fourier transformations are applicable, e.g., in case of truncated responses.

We first introduce, as for standard VF [6], a scalar weight function

$$\sigma_k(s) = 1 + \sum_{n=1}^{N_k} \frac{r_{k,n}}{s - q_{k,n}} = \frac{\prod_{n=1}^{N_k}(s - z_{k,n})}{\prod_{n=1}^{N_k}(s - q_{k,n})} \tag{9}$$

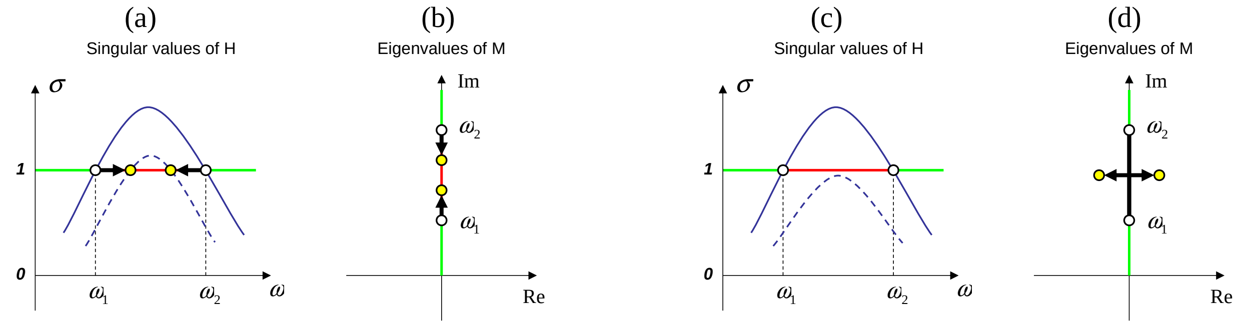

Figure 1: Perturbation of eigenvalues of Hamiltonian matrix $\mathcal{M}$ (b) and its induced effect on the singular values of the macromodel (a). Iterative application leads to displacement of the eigenvalues off the imaginary axis (d) and consequently to passivity enforcement (c).

with fixed (initial) poles $\{q_{k,n}\}$ and unknown residues $\{r_{k,n}\}$. This function is used to enforce the following condition,

$$\sigma_k(s)\boldsymbol{H}_k(s) \simeq \boldsymbol{M}_{k,\infty} + \sum_{n=1}^{N_k} \frac{\boldsymbol{M}_{k,n}}{s - q_{k,n}} \,. \tag{10}$$

Since the right-hand-side has the same poles $\{q_{k,n}\}$ as the weight function, a cancellation between the zeros $\{z_{k,n}\}$ of $\sigma_k(s)$ and the poles $\{p_{k,n}\}$ of $\boldsymbol{H}_k(s)$ must occur. This condition allows to estimate these poles by solving (10) for the unknown residues $\{r_{k,n}\}$, computing the zeros $\{z_{k,n}\}$ using standard techniques [6], and by enforcing $p_{k,n} = z_{k,n}$. This procedure, named *pole relocation*, avoids use of ill-conditioned nonlinear least squares algorithms. Also, the poles relocation can be iterated using the estimated poles as starting poles for the new iteration.

We reformulate the VF condition (10) in time domain by applying it to the input signal $X_{j_k}(s)$ and by using inverse Laplace transform. We get

$$\boldsymbol{y}_k(t) \simeq \boldsymbol{M}_{k,\infty} x_{j_k}(t) + \sum_{n=1}^{N_k} \boldsymbol{M}_{k,n} x_{j_k,n}(t) - \sum_{n=1}^{N_k} r_n \boldsymbol{y}_{k,n}(t) \,, \tag{11}$$

where the transient waveforms

$$x_{j_k,n}(t) = \int_0^t e^{q_{k,n}(t-\tau)} x_{j_k}(\tau) d\tau \,, \qquad \boldsymbol{y}_{k,n}(t) = \int_0^t e^{q_{k,n}(t-\tau)} \boldsymbol{y}_k(\tau) d\tau \tag{12}$$

are convolutions resulting from inverse Laplace transform of each partial fraction in the expansions (9)-(10). These waveforms are easily obtained by applying a suitable discretization of the convolution integrals. We remark that due to the exponential nature of the convolution kernels, a fast implementation based on recursive convolutions, i.e., digital IIR filtering, is convenient [2].

The condition (11) is enforced in least squares sense using raw and filtered input/output sequences. Solution of this linear system returns the residues $\{r_{k,n}\}$ of the weight function, which in turn are used as in standard VF to compute its zeros $\{z_{k,n}\}$, and consequently the poles $\{p_{k,n}\}$ of the sought approximation. Once these poles are known, the residues $\boldsymbol{R}_{k,n}$ and the direct coupling matrix $\boldsymbol{H}_{k,\infty}$ in (8) are computed by solving another linear least squares problem,

$$\boldsymbol{y}_{k,n}(t) \simeq \boldsymbol{H}_{k,\infty} \widehat{x}_{j_k}(t) + \sum_{n=1}^{N_k} \boldsymbol{R}_{k,n} \widehat{x}_{j_k,n}(t) \,, \tag{13}$$

with $\widehat{x}_{j_k,n}(t)$ defined as in (12) with $\{q_{k,n}\}$ replaced by the estimated poles $\{p_{k,n}\}$.

4 Global State-Space Realization

This section describes how to generate a global macromodel from the set of partial macromodels in (13). The first step is to convert the poles-residues representation of (13) into an equivalent state-space realization

$$\left\{ \begin{array}{rcl} \frac{d}{dt}\boldsymbol{w}_k(t) & = & \boldsymbol{A}_k \boldsymbol{w}_k(t) + \boldsymbol{B}_k x_{j_k}(t) \\ \boldsymbol{y}_k(t) & = & \boldsymbol{C}_k \boldsymbol{w}_k(t) + \boldsymbol{D}_k x_{j_k}(t) \end{array} \right. \tag{14}$$

This operation is standard and is not further commented here. A straightforward derivation shows that a global state-space representation in the form

$$\left\{ \begin{array}{rcl} \frac{d}{dt}\boldsymbol{w}(t) & = & \boldsymbol{A}\boldsymbol{w}(t) + \boldsymbol{B}\boldsymbol{x}(t) \\ \boldsymbol{y}(t) & = & \boldsymbol{C}\boldsymbol{w}(t) + \boldsymbol{D}\boldsymbol{x}(t) \end{array} \right. \tag{15}$$

can be obtained by defining the global state-space matrices as

$$\begin{array}{rcl} \boldsymbol{A} & = & \mathrm{blockdiag}\{\boldsymbol{A}_k\} \\ \boldsymbol{B} & = & \mathrm{blockdiag}\{\boldsymbol{B}_k\}\,\mathrm{stack}\{\boldsymbol{P}_k\} \\ \boldsymbol{C} & = & \mathrm{stack}\{\boldsymbol{Q}_k\}^T\,\mathrm{blockdiag}\{\boldsymbol{C}_k\} \\ \boldsymbol{D} & = & \mathrm{stack}\{\boldsymbol{Q}_k\}^T\,\mathrm{blockdiag}\{\boldsymbol{D}_k\}\,\mathrm{stack}\{\boldsymbol{P}_k\} \end{array} \tag{16}$$

where the $\mathrm{blockdiag}\{\cdot\}$ and $\mathrm{stack}\{\cdot\}$ tile their arguments as block matrices in a diagonal and column form, respectively. As a result, the final macromodel has a quite sparse representation since by construction its state-space matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are block-diagonal. Note that in case of complex poles a real state-space form is easily obtained by using 2×2 building blocks in $\boldsymbol{A}$.

5 Passivity Enforcement

The final step in the construction of the macromodel is the check of its passivity and, if needed, the compensation of any passivity violation. This is achieved by a spectral perturbation of the Hamiltonian matrix associated to the macromodel. Such matrix has different forms according to the particular representation of the input-output relations. For presentation purposes, we assume here a scattering representation, although the same results can be applied also for admittance, impedance, or hybrid form [5].

Macromodel passivity is guaranteed if the transfer (scattering) matrix is unitary bounded at all frequencies, or equivalently, if the set $\sigma(\boldsymbol{H}(j\omega))$ of all its singular values is not larger than one at any frequency. Instead of using a possibly erroneous and time-comsuming frequency-sweep passivity test, we use a purely algebraic criterion based on the Hamiltonian matrix. This matrix is defined for the scattering representation as

$$\mathcal{M} = \begin{pmatrix} \boldsymbol{A} - \boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{D}^T\boldsymbol{C} & -\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T \\ \boldsymbol{C}^T\boldsymbol{S}^{-1}\boldsymbol{C} & -\boldsymbol{A}^T + \boldsymbol{C}^T\boldsymbol{D}\boldsymbol{R}^{-1}\boldsymbol{B}^T \end{pmatrix}, \tag{17}$$

with $\boldsymbol{R} = (\boldsymbol{D}^T\boldsymbol{D} - \boldsymbol{I})$ and $\boldsymbol{S} = (\boldsymbol{D}\boldsymbol{D}^T - \boldsymbol{I})$. Note that this matrix is constructed from the state-space matrices of the macromodel. Under some technical conditions that are always verified for the particular form of macromodel here considered [5, 1], the existence of purely imaginary eigenvalues of this matrix can be related to the existence of passivity violations of the macromodel. In particular, one of the singular values of the macromodel reaches the threshold value $\gamma = 1$ at a frequency ω_0 if and only if $j\omega_0$ is an eigenvalue of $\mathcal{M}$. This condition can be applied to detect very precisely the frequency bands where passivity violations occur. Details of this procedure can be found in [5, 4].

If some violations are found, a procedure based on first-order perturbations is applied in order to eliminate them. This is achieved by perturbing the imaginary eigenvalues of the Hamiltonian matrix and forcing them to move off the imaginary axis, thus insuring passivity. Each imaginary eigenvalue is displaced by an amount $\Delta\omega_i$, determined on the basis of the

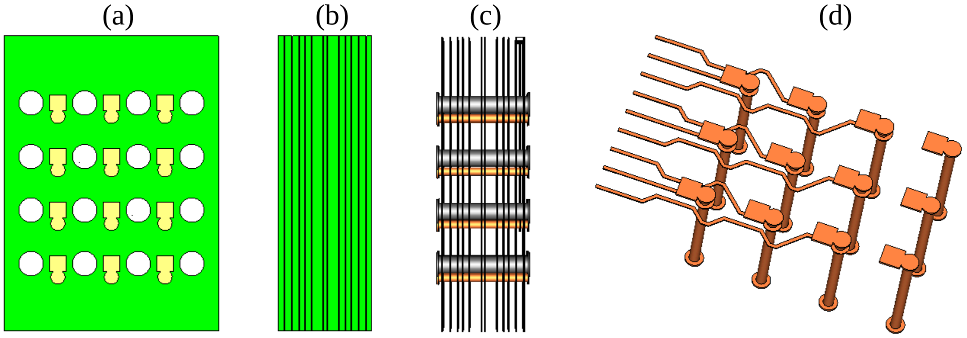

Figure 2: Multiple via structure on a printed circuit board: (a) top view, (b) side view, (c) side view with dielectric removed, and (d) perspective view of signal vias and traces only.

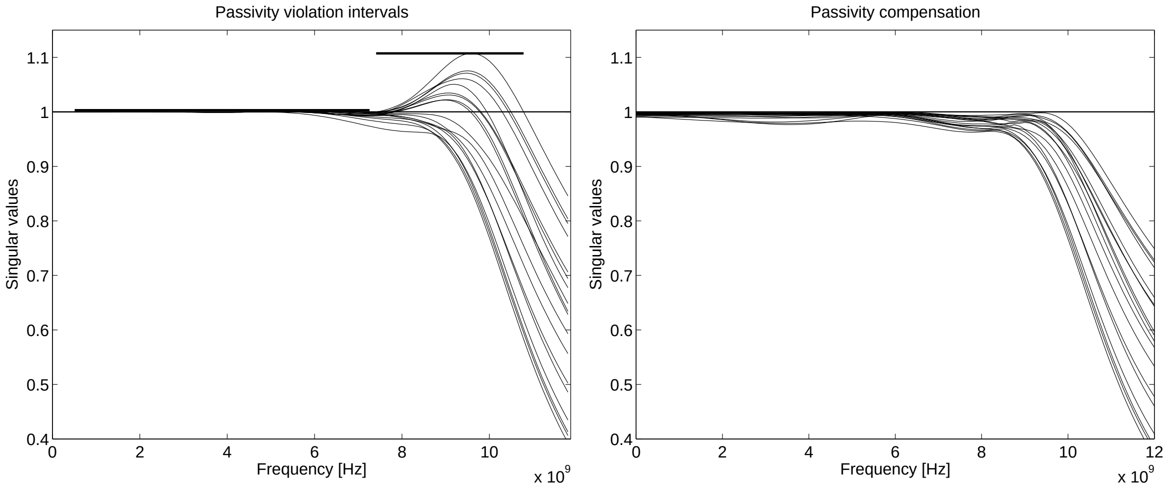

Figure 3: Passivity compensation for the multiple via structure of Fig. 2. The singular values are plotted versus frequency for the non-passive macromodel (left panel) and for the passive macromodel (right panel).

location of the violation bands. This displacement is enforced by computing a suitable correction term $\boldsymbol{dC}$ for the state matrix $\boldsymbol{C}$. Using a first-order analysis this correction term can be related linearly to the the set of eigenvalue perturbations $\Delta\omega_i$, leading to a simple linear least squares system. The details of this passivity compensation procedure can be found in [5, 4]. Figure 1 illustrates the perturbation in a graphical form.

6 Numerical Results

The macromodeling process is illustrated using the structure depicted in Fig. 2. A multilayer PCB with several power and ground planes is crossed by several via interconnects throughout its cross-section. The vias serve as interface between the traces on one of the signal layers and a connector to be mounted on top of the board. Here we consider only a matrix of $9 = 3 \times 3$ via interconnects (the other vias are not connected, see Fig. 2), and we want to derive a suitable lumped macromodel for the resulting structure having $p = 18$ ports. To this end, the structure has been meshed and analyzed with a full-wave transient electromagnetic solver based on Finite Integration [7]. All ports are defined using a 50Ω reference load. The raw dataset consists of 18×18 transient scattering responses due to Gaussian pulse excitation having a 6 GHz frequency bandwidth. This set of responses has been processed by the Time-Domain Vector Fitting algorithm. An excellent approximation was obtained using only $n_p = 6$ poles for each element of the transfer matrix. Although very accurate, the resulting state-space realization is not passive, as depicted in the left panel of Fig. 3.

The figure shows the presence of many crossings of the critical level $\gamma = 1$. For this specific case, the number of singular values exceeding this threshold reaches 18 (i.e., all singular values) in a small frequency interval. This is in fact a quite a challenging example for any passivity compensation algorithm. Also, this example is illustrative of a typical problem occurring in the generation of rational macromodels. Due to the presence of a pair of complex poles slightly outside the frequency band of the excitation pulse, the passivity violation outside this bandwidth can be significant. Conversely, the in-band behavior is characterized by a very small violation (the largest in-band singular value is bounded by $\sigma_{\max} \simeq 1.003$. It should be kept in mind that a bandlimited excitation may only lead to the identification of a macromodel that is accurate in the tested frequency interval.

The Hamiltonian-based compensation algorithm was applied to compute a passive approximation. A total number of 32 iterations were necessary for the passivity compensation, with a resulting perturbation on the state matrix such that

$$||\boldsymbol{dC}||_F = 0.049\, ||\boldsymbol{C}||_F \,. \tag{18}$$

The distribution of singular values of the passive macromodel is depicted in the right panel of Fig. 3. We report in Fig. 4 some of the transient port responses of the two macromodels. The accuracy is excellent also in this case. This example shows that compensation of significant out-of-band passivity violation does not affect the accuracy of the macromodel, provided that it is excited by signals having most of their spectral components within the design bandwidth.

Acknowledgements. The Author is grateful to Dr. Erich Klink of IBM for providing the data used in the numerical example. This work is supported in part by the Italian Ministry of University (PRIN grant #2002093437), and in part by CERCOM, Politecnico di Torino.

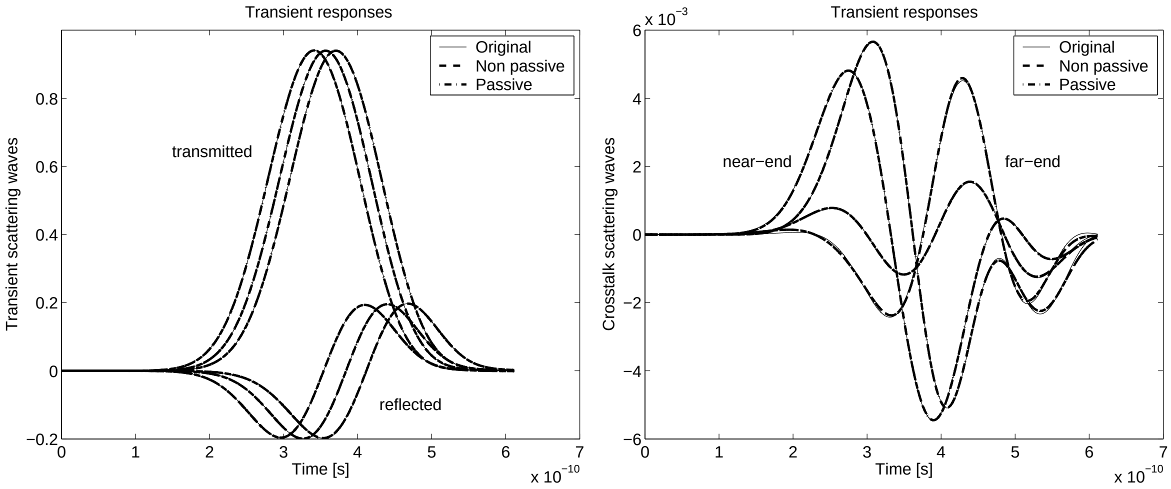

Figure 4: Selected transient responses for multiple via structure of Fig. 2. The left panel shows transmitted and reflected waves for three different excited vias in a row. The right panel shows near-end and far-end crosstalk waveforms for the same three vias.

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